

Pair condensation of polarized fermions in the BCS-BEC crossover

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Abstract. We investigate a two-component Fermi gas with unequal spin populations along the BCS-BEC crossover. By using the extended BCS equations and the concept of off-diagonal-long-range-order we derive a formula for the condensate number of Cooper pairs as a function of energy gap, average chemical potential, imbalance chemical potential and temperature. Then we study the zero-temperature condensate fraction of Cooper pairs by varying interaction strength and polarization, finding a depletion of the condensate fraction by increasing the population imbalance. We also consider explicitly the presence of an external harmonic confinement and we study, within the local-density approximation, the phase separation between superfluid and normal phase regions of the polarized fermionic cloud. In particular, we calculate both condensate density profiles and total density profiles from the inner superfluid core to the normal region passing for the interface, where a finite jump in the density is a clear manifestation of this phase-separated regime. Finally, we compare our theoretical results with the available experimental data on the condensate fraction of polarized ⁶Li atoms [Science **311**, 492 (2006)]. These experimental data are in reasonable agreement with our predictions in a suitable range of polarizations, but only in the BCS side of the crossover up to unitarity.

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1. Introduction

The experimental realization of the predicted crossover from the Bardeen-Cooper-Schrieffer (BCS) state of weakly bound Fermi pairs to the Bose-Einstein condensate (BEC) of molecular dimers [1, 2, 3, 4, 5, 6, 7, 8, 9] is one of the most important achievements of atomic physics over the past years. In two experiments [7, 9] the condensate fraction of Cooper pairs [10], which is directly related to the off-diagonal-long-range-order of the two-body density matrix of fermions [11, 12], has been investigated with two-hyperfine-component Fermi vapors of ^6Li atoms in the BCS-BEC crossover. The experimental results exhibit a quite good agreement with mean-field theoretical predictions [13, 14] and Monte-Carlo simulations at zero temperature [15]. Moreover, the condensate fraction is a relevant quantity also in understanding the BCS-BEC crossover in the presence of spin-orbit couplings [16, 17], in the case of a narrow resonance [18], and also for the two-dimensional Fermi superfluid [19, 20, 21].

A very interesting extension of these studies is the analysis of two-component trapped Fermi gases with polarization, i.e. with a population imbalance between the two hyperfine components. Many efforts have been devoted to this topic both theoretically [22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41] and experimentally [42, 43, 44]. The polarized Fermi gas is characterized by a far richer phase diagram than the equal spin-population case. Such a physical system, in fact, exhibits a quantum phase transition between the superfluid (SF) and normal (N) states, and it has been predicted to possess exotic SF phases such as the inhomogeneous Fulde-Ferrell-Larkin-Ovchinnikov state [45, 46], and a phase-separated (PS) regime [22, 23, 31, 33, 27, 35, 37, 40, 41, 42, 43] where a normal phase coexists with a superfluid state. Zwierlein *et al* [42] have measured the condensate fraction of Cooper pairs as a function of the polarization both in the BCS region and on the BEC side of the crossover. However, on the theoretical side, it is still missing a systematic analysis of the condensate fraction in the imbalanced case.

In this paper we provide a theoretical investigation of pair condensation of polarized fermions in the BCS-BEC crossover. By using a path-integral formalism we derive for the polarized superfluid Fermi gas an analytical formula for the condensate fraction of Cooper pairs, which is then studied with generalized BCS equations, at zero temperature and in the uniform case, as a function of the dimensionless interaction parameter $y = 1/(k_F a_s)$, with k_F the Fermi wave vector and a_s the s-wave fermion-fermion scattering length, and for different polarizations. From this analysis we find that an increase of the population imbalance gradually produces a depletion of the condensate fraction, which is shown to decrease linearly with the polarization on the BEC side of the resonance. Finally, we consider the inclusion of a realistic axially-symmetric harmonic potential. Within the local density approximation we analyze the behavior of both total density profile and condensate density profile for different scattering lengths finding a finite jump in the profiles, which is a clear signature of the phase boundary between the superfluid phase (in the inner core) and normal phase (in the outer region) where pair

condensation is absent. These results are compared with the experimental ones [42].

2. Extended BCS equations for the polarized Fermi gas

We consider a two-spin-component ($\sigma = \uparrow, \downarrow$) Fermi gas with unequal spin populations having the same mass m . The fermions are supposed to interact via a contact potential. In the absence of external confinement, our system can be described by the following (Euclidean) Lagrangian density [47, 48]:

$$\begin{aligned} \mathcal{L} = & \sum_{\sigma=\uparrow,\downarrow} \bar{\psi}_\sigma(\mathbf{r}, \tau) \left(\hbar \frac{\partial}{\partial \tau} - \hbar^2 \frac{\nabla^2}{2m} - \mu_\sigma \right) \psi_\sigma(\mathbf{r}, \tau) \\ & + g \bar{\psi}_\uparrow(\mathbf{r}, \tau) \bar{\psi}_\downarrow(\mathbf{r}, \tau) \psi_\downarrow(\mathbf{r}, \tau) \psi_\uparrow(\mathbf{r}, \tau) . \end{aligned} \quad (1)$$

Here $\psi_\sigma, \bar{\psi}_\sigma$ are the Grassmann field variables; $g < 0$ is the strength coupling of the fermion-fermion attractive contact-potential interaction; μ_σ is the chemical potential of the component σ . In the rest of the paper for simplicity we set $\hbar = k_B = 1$, with \hbar the reduced Planck constant and k_B the Boltzmann constant. We define the average chemical potential μ and the imbalance chemical potential ζ as the half-sum and the half-difference between the two chemical potentials, respectively, i.e.

$$\mu = \frac{\mu_\uparrow + \mu_\downarrow}{2} \quad \zeta = \frac{\mu_\uparrow - \mu_\downarrow}{2} . \quad (2)$$

In the following, without loss of generality, we assume that the \uparrow species is the majority component.

We are interested in studying the thermodynamics of the system at temperature T in a volume V . All the thermodynamical properties can be inferred from the partition function Z , which in the path integral formalism reads

$$Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S[\psi, \bar{\psi}]} , \quad (3)$$

where

$$S[\psi, \bar{\psi}] = \int_0^\beta d\tau \int_V d^3\mathbf{r} \mathcal{L} \quad (4)$$

is the Euclidean action with $\beta = 1/T$ [47, 48].

From the partition function, one can calculate the number of fermions with spin σ given by

$$N_\sigma = \frac{1}{\beta} \frac{\partial}{\partial \mu_\sigma} \ln Z . \quad (5)$$

The thermodynamics of the system can be studied by decoupling the Grassmann field-quartic interaction term in the second row of Eq. (1) by means of a Hubbard-Stratonovich transformation. The resulting complex auxiliary field $\Delta(\mathbf{r}, \tau)$ is then expanded around its saddle point value $\Delta(\mathbf{r}, \tau) = \Delta_0 + \delta(\mathbf{r}, \tau)$. At this stage we treat our system by using the mean-field (MF) approximation which consists in neglecting the fluctuations $\delta(\mathbf{r}, \tau)$. In this approximation, Δ_0 is the real MF order parameter [47, 48].

In the Nambu-Gorkov formalism [49], our theory is described by the following 2×2 momentum-space Green function:

$$\tilde{G}^{-1} = \begin{pmatrix} i\omega_n + \frac{k^2}{2m} - \mu - \zeta & \Delta_0 \\ \Delta_0 & i\omega_n - \frac{k^2}{2m} + \mu - \zeta \end{pmatrix} \quad (6)$$

from which it is immediate to get the energy spectrum of the Bogoliubov excitations:

$$E_{\mathbf{k}}^{\pm} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta_0^2} \pm \zeta, \quad (7)$$

where $\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu = \frac{k^2}{2m} - \mu$ and Δ_0 is the familiar energy gap of fermionic elementary excitations. The sign plus (minus) in Eq. (7) holds for \uparrow (\downarrow) component. The fermionic fields $\bar{\psi}(\mathbf{r}, \tau)$ and $\psi(\mathbf{r}, \tau)$ can be integrated out, and the summation over Matsubara frequencies yields the following effective action:

$$S_e = -\beta V \frac{\Delta_0^2}{g} + \beta \sum_{\mathbf{k}} \xi_{\mathbf{k}} - \sum_{\mathbf{k}} \ln(2 \cosh(\beta\zeta) + 2 \cosh(\beta E_{\mathbf{k}})) \quad (8)$$

with $E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta_0^2}$. From the effective action (8) we achieve the extended BCS (EBCS) equations at finite temperature [47, 48]:

$$\frac{1}{g} = \frac{1}{V} \sum_{\mathbf{k}} \frac{1}{E_{\mathbf{k}}} \frac{\sinh(\beta E_{\mathbf{k}})}{2 \cosh(\beta\zeta) + 2 \cosh(\beta E_{\mathbf{k}})}, \quad (9)$$

$$N = \sum_{\mathbf{k}} \left(1 - \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}} \frac{\sinh(\beta E_{\mathbf{k}})}{\cosh(\beta\zeta) + \cosh(\beta E_{\mathbf{k}})} \right), \quad (10)$$

$$N_{\uparrow} - N_{\downarrow} = 2 \sum_{\mathbf{k}} \frac{\sinh(\beta\zeta)}{\cosh(\beta\zeta) + \cosh(\beta E_{\mathbf{k}})}. \quad (11)$$

The first of the three above equations is the gap equation, Eq. (10) is the number equation, and, finally, Eq. (11) is the equation for the population imbalance.

The gap equation (9) needs to be regularized, and this can be done according to the following prescription (see, for instance, [2]):

$$\frac{m}{4\pi a_s} = -\frac{1}{g} + \frac{1}{V} \sum_{\mathbf{k}} \frac{1}{2\epsilon_{\mathbf{k}}}, \quad (12)$$

where a_s is the s-wave scattering length. In such a way one attains the regularized gap equation

$$\frac{m}{4\pi a_s} = \frac{1}{2V} \sum_{\mathbf{k}} \left(\frac{1}{\epsilon_{\mathbf{k}}} - \frac{1}{E_{\mathbf{k}}} \frac{\sinh(\beta E_{\mathbf{k}})}{\cosh(\beta\zeta) + \cosh(\beta E_{\mathbf{k}})} \right). \quad (13)$$

Finally, we calculate the grand potential

$$\Omega = -\frac{1}{\beta} \ln Z.$$

By using in this equation, Eqs. (3) and (8), the mean-field version of Ω is given by

$$\Omega = -V \frac{\Delta_0^2}{g} + \sum_{\mathbf{k}} (\xi_{\mathbf{k}} - E_{\mathbf{k}}) - \frac{1}{\beta} \sum_{\mathbf{k}} [\ln(1 + e^{-\beta(E_{\mathbf{k}} - \zeta)}) + \ln(1 + e^{-\beta(E_{\mathbf{k}} + \zeta)})]. \quad (14)$$

In the limit of zero temperature ($T = 0$, i.e. $\beta \rightarrow \infty$), the MF grand potential (14) reads

$$\Omega_{T=0} = \sum_{\mathbf{k}} (\xi_{\mathbf{k}} - E_{\mathbf{k}}) - V \frac{\Delta_0^2}{g} + \sum_{k_- < |\mathbf{k}| < k_+} (E_{\mathbf{k}} - \zeta) , \quad (15)$$

where $k_- = \sqrt{2m} \sqrt{\max(\mu - \sqrt{\zeta^2 - \Delta_0^2}, 0)}$ and $k_+ = \sqrt{2m} \sqrt{\max(\mu + \sqrt{\zeta^2 - \Delta_0^2}, 0)}$.

The first two terms of Eq. (15) are the same as in the unpolarized two-component Fermi gas, while the last one describes the polarized case and contributes to the physics of the system provided that $\zeta \geq \Delta_0$.

As discussed for the two-dimensional case in [50], also in the 3D case, in the zero-temperature limit, the state of the system can be thought of as a superfluid in which the particles with momenta $|\mathbf{k}| \in [k_-, k_+]$ contribute as normal phase particles.

3. Condensate of Cooper pairs

This section is devoted to study the condensate fraction, that is the ratio

$$\phi \equiv \frac{N_0}{N} \quad (16)$$

where N_0 is the condensate number of Cooper pairs and N is the total number of fermions. N_0 is the largest eigenvalue of the two-body density matrix [51], and a finite value of ϕ in the thermodynamic limit ($N, V \rightarrow \infty$ and $n = N/V$ constant) implies off-diagonal long-range order [11]. As a result of spontaneous breaking of $U(1)$ symmetry N_0 is given by [13, 51]

$$N_0 = \sum_{\sigma\sigma'} \int d^3\mathbf{r} \int d^3\mathbf{r}' |\langle \psi_{\sigma}(\mathbf{r}) \psi_{\sigma'}(\mathbf{r}') \rangle|^2 , \quad (17)$$

and it can be calculated by using the path integral formalism. From the Nambu-Gorkov Green function we find

$$N_0 = \frac{1}{\beta^2} \sum_{\mathbf{p}} \sum_n \sum_m G_{21}(\mathbf{p}, i\omega_n) G_{12}(\mathbf{p}, i\omega_m) , \quad (18)$$

where G_{21} and G_{12} are obtained by inverting Eq. (6):

$$G_{12}(\mathbf{k}, i\omega_n) = -\frac{\Delta_0}{(i\omega_n - \zeta)^2 - \xi_{\mathbf{k}}^2 - |\Delta_0|^2} = G_{21}(\mathbf{k}, i\omega_n) . \quad (19)$$

After performing the summation over the Matsubara fermionic frequencies, one gets the condensate number (18) as a function of the chemical potential μ (the left formula of Eq. (2)) and of the MF order parameter Δ_0

$$N_0 = \sum_{\mathbf{k}} \frac{\Delta_0^2}{4E_{\mathbf{k}}^2} \left(\frac{1}{2} \tanh\left(\frac{\beta}{2}(E_{\mathbf{k}} + \zeta)\right) + \frac{1}{2} \tanh\left(\frac{\beta}{2}(E_{\mathbf{k}} - \zeta)\right) \right)^2 . \quad (20)$$

This formula is the main analytical result of the paper: it gives the condensate number of Cooper pairs in term of the energy gap Δ_0 , average chemical potential μ , imbalance chemical potential ζ , and temperature T .

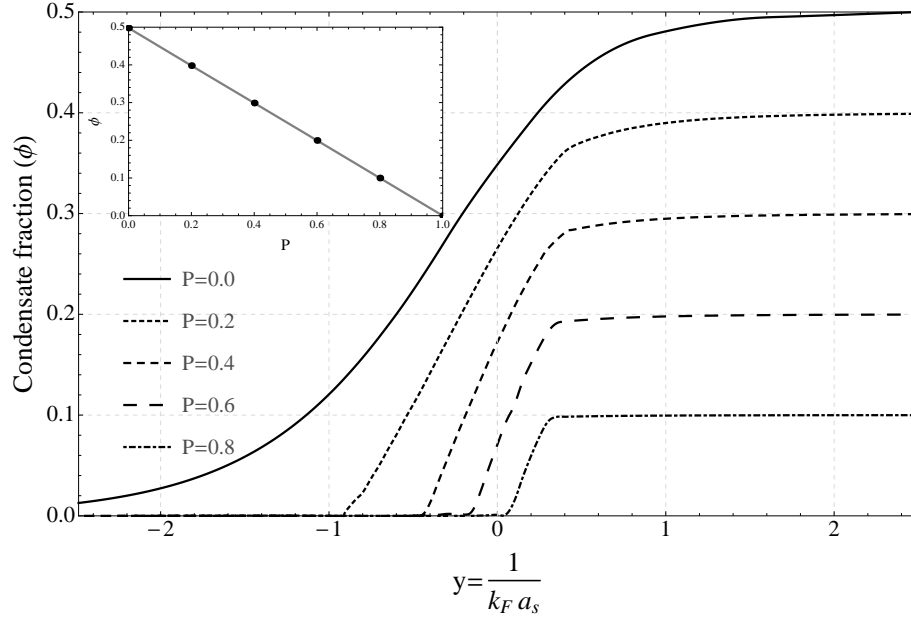


Figure 1: The condensate fraction $\phi = N_0/N$ as a function of the inverse dimensionless interaction parameter $y = 1/(k_F a_s)$ for different values of the polarization. In the inset ϕ as a function of the polarization $P = (N_\uparrow - N_\downarrow)/(N_\uparrow + N_\downarrow)$ for $y = 2$.

We are interested in the zero-temperature analysis of our system. In this limit, i.e. $\beta \rightarrow \infty$, the condensate fraction is calculated as a momentum space sum. In the balanced case the sum runs over all momenta, while for a polarized gas the Bogoliubov excitations with $|\mathbf{k}| \in [k_-, k_+]$ do not contribute to the condensate fraction, since the energy of the \downarrow fermions $E_{\mathbf{k}} - \zeta$ is ≤ 0 :

$$N_0 = \sum_{|\mathbf{k}| \notin [k_-, k_+]} \frac{\Delta_0^2}{4E_{\mathbf{k}}^2}. \quad (21)$$

We use this formula for N_0 and the zero-temperature version of Eqs. (9)-(11) to calculate numerically the condensate fraction ϕ as a function of the dimensionless interaction parameter

$$y = \frac{1}{k_F a_s}, \quad (22)$$

with $k_F = (3\pi n)^{1/3}$ the Fermi wavenumber, and of the polarization

$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}. \quad (23)$$

The results of these calculations are reported in Fig. 1, where we plot the condensate fraction $\phi = N_0/N$ as a function of the inverse dimensionless interaction parameter y for different values of the polarization P .

In Fig. 1 the solid line, corresponding to $P = 0$, is exactly equal to the solid one reported in Fig. 1 of Ref. [13]. The other curves, corresponding to different finite values of P , are instead new theoretical results. From Fig. 1 one observes the expected behavior in the deep-BCS regime ($y \ll -1$), that is a weak superfluidity which is destroyed as

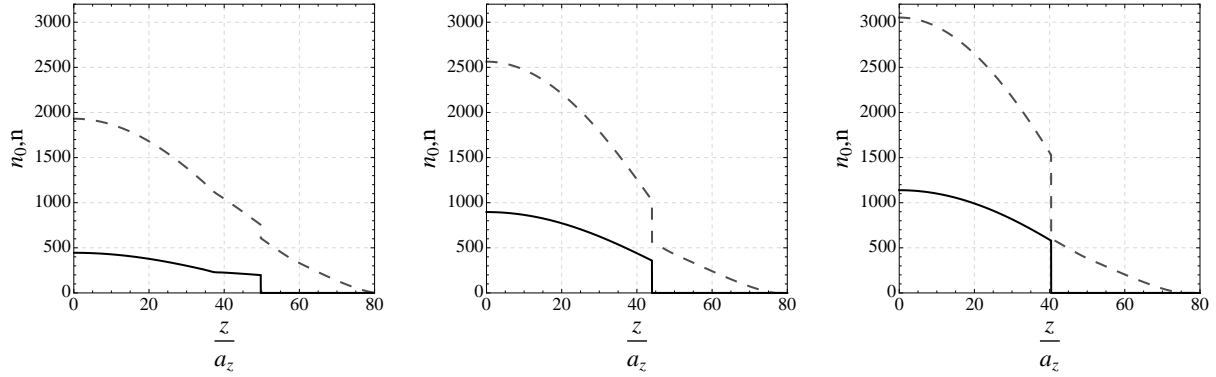


Figure 2: Polarized Fermi gas in the harmonic trap. The condensate density profile $n_0(z)$ (solid line) and total density profile $n(z)$ (dashed line) in the axial direction z for three different scattering lengths. From left to right: $y = -0.44$, $y = 0.0$, $y = 0.11$, where $y = 1/(k_F a_s)$ with $k_F = (3\pi^2 n(\mathbf{0}))^{1/3}$ and $n(\mathbf{0})$ the total density at the center of the trap. Number of atoms $N = 2.3 \cdot 10^7$ and polarization $P = (N_\uparrow - N_\downarrow)/N = 0.2$. Here $a_z = 1/\sqrt{m\omega_z}$ is the characteristic length of the axial harmonic confinement.

soon as the polarization P becomes finite. On the other hand, the results in the deep-BEC regime ($y \gg 1$) are easily interpreted considering that a number of fermions equal to $N_{pairs} = 2 \min(N_\uparrow, N_\downarrow)$ will produce $N_{pairs}/2$ boson-like bound pairs for sufficiently attractive interactions, while the remaining

$$N_{normal} = N - N_{pairs} = N_\uparrow - N_\downarrow = NP$$

fermions give a normal-state Fermi gas. The former kind of particles contribute to the condensate fraction, while the latter type does not. Hence, by noting that N_{normal} is proportional to the polarization, and by also noting that $N_{pairs} = 2N - 2N_{normal}$, we expect to observe that in the deep BEC regime $N_{pairs} \propto (1 - P) \propto \phi$, as verified in the inset of Fig. 1.

To conclude this section, we observe that our mean-field approach could be extended by determining the magnitude of the contribution from Gaussian fluctuations by following [53] and thus adding a beyond-mean-field correction to the condensate number given by:

$$N_0^{(2)} = \frac{1}{\beta} \sum_{\mathbf{p}, i\omega_n} \text{Tr} (\tau_3 G(\mathbf{p}, i\omega_n) \Sigma(\mathbf{p}, i\omega_n) G(\mathbf{p}, i\omega_n)), \quad (24)$$

where τ_3 is the Pauli matrix in the z direction, $\Sigma(\mathbf{p}, i\omega_n)$ is an effective self-energy, and the trace is meant to be taken in the Nambu space [53].

As discussed in [53], Gaussian fluctuations play a key role in determining the critical temperature and the finite-temperature properties of a Fermi gas. However, in the zero-temperature limit we are investigating in the present paper, the effects of Gaussian fluctuations on the condensate fraction are not detectable given the experimental sensitivities of the data we are comparing our theory with.

4. External harmonic confinement and local-density approximation

Usually a polarized Fermi gas is experimentally investigated by confining it in a suitable trapping potential $V(\mathbf{r})$ given by the axially-symmetric harmonic trap

$$V(\mathbf{r}) = \frac{m}{2} [\omega_{\perp}^2(x^2 + y^2) + \omega_z^2 z^2] \quad (25)$$

where ω_{\perp} and ω_z are the transverse and axial trapping frequencies, respectively.

We treat the presence of the external confinement by using the local density approximation (LDA). In this approximation, a local chemical potential

$$\mu_{\sigma} \rightarrow \mu_{\sigma}(\mathbf{r}) = \mu_{\sigma} - V(\mathbf{r}) \quad (26)$$

is introduced and the system is treated as locally uniform. As a result, the EBCS gap equation must be solved at each point of the space for a spatially-dependent gap $\Delta(\mathbf{r})$ for a given scattering length a_s .

The zero-temperature free energy

$$F = \Omega_{T=0} + \sum_{\sigma} \mu_{\sigma} N_{\sigma} , \quad (27)$$

with $\Omega_{T=0}$ given by Eq. (15), exhibits two minima, one in correspondence to a non-zero order parameter that we denote by $\Delta_0(\mathbf{r})$ (superfluid phase), and the other corresponding to $\Delta_0 = 0$ (normal phase). On the interplay between these two minima relies the phase separation in an unbalanced trapped Fermi gas. By requiring that the superfluid and the normal states have locally the same free energy, one finds a critical value $\Delta_{0,c}$ of the energy gap [37]. In the spatial region of the trap where $\Delta_0(\mathbf{r}) > \Delta_{0,c}$, the system is superfluid, while in the spatial region where $\Delta_0(\mathbf{r}) < \Delta_{0,c}$ the system is normal.

In Fig. 2, we report the density profile $n(z) = n_{\uparrow}(x=0, y=0, z) + n_{\downarrow}(x=0, y=0, z)$ in the axial direction z , and the condensate density profile, $n_0(z) = \int n_0(x=0, y=0, z)$ in the axial direction z for three values of the scattering length a_s and fixed total number $N = 2.3 \cdot 10^7$ of atoms and polarization $P = 0.2$. From the figure one finds that the higher-density superfluid regions are located at the center of the trap, while the lower-density fully polarized normal state are expelled outside (see also [41]). In accordance with other theoretical works [36, 37] and with experimental data [54], Fig. 2 shows that the superfluid phase ends abruptly at a critical distance z_c from the center of the cloud: the condensate fraction jumps from a finite value to zero. This effect is a clear manifestation of the phase separation. Note that Monte Carlo simulations suggest that the LDA boundary of the superfluid phase is slightly overestimated [55, 56].

5. Interpretation of the results: theory vs. experiment

The experiment of Ref. [42] with trapped ^6Li atoms addresses the problem of measuring the condensate fraction ϕ of trapped interacting fermions near the unitarity limit as a function of the polarization P . We attempt a comparison of the computed total condensed fraction with the experimental data at the lowest temperature $T = 300$

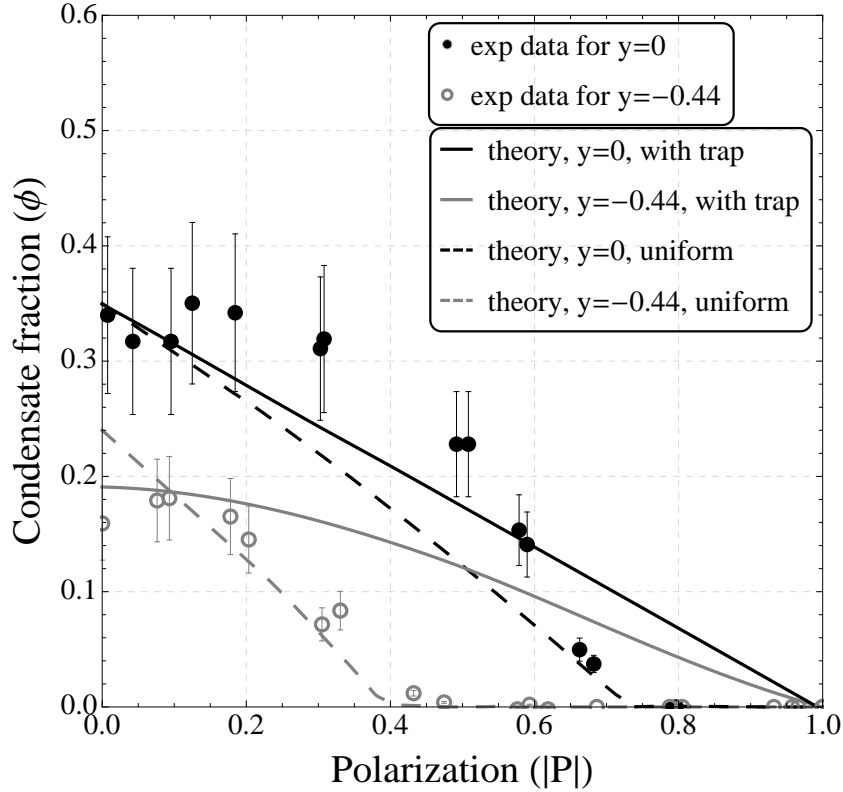


Figure 3: Condensate fraction ϕ as a function of the absolute value of the polarization $|P|$ for two values of the dimensionless interaction parameter $y = 1/k_F a_s$: $y = -0.44$ (open circles) and $y = 0.0$ (filled circles). Circles with error bars are experimental data of ^6Li atoms taken from Ref. [42]. Solid lines are our theoretical calculations for the trapped system. For completeness, we report also our theoretical results of the uniform system (dashed lines).

nKelvin [42]. In the experiment the scattering length is tuned by means of an external magnetic tuned across a Feshbach resonance. Following Ref. [52], the scattering length a_s as a function of the magnetic field B near the Feshbach resonance is given by

$$a_s = a_b [1 + \alpha(B - B_0)] \left[1 + \frac{B_r}{B - B_0} \right], \quad (28)$$

where $B_0 = 83.4149$ mT, $a_b = -1405 \cdot 0.53 \cdot 10^{-10}$ m, $B_r = 30.0$ mT, and $\alpha = 0.0040$ (mT) $^{-1}$. The measured condensed fraction at the lowest temperature is reported in Fig. 3 as circles and squares while our theoretical results are the solid lines. For the sake of completeness, we report also the theoretical calculations for the uniform system (dashed line)).

In Fig. 3 it is shown ϕ as a function of P for two values of $y = 1/k_F a_s$: $y = -0.44$ (open circles) and $y = 0.0$ (filled circles). The error bars in the experimental data for $\phi = \frac{N_0}{N}$ have been calculated by using the standard error propagation formula considering the 20% error on the determination of N , as reported in Ref.[42]. The figure clearly shows that, in the BCS side of the crossover up to the unitarity limit ($y \leq 0$), the general trend of the experimental data agrees with our EBCS calculation

but the agreement deteriorates by increasing the polarization P .

It is important to stress that in the BEC regime ($y > 0$) the experimental data [42] are strongly affected by inelastic losses and they show a rapid drop of the measured ϕ as a function of y at fixed polarization P . This is clearly incompatible (see Fig. 1) with our calculations. For this reason we restrict our comparison in Fig. 3 to the BCS and unitarity regimes.

By analyzing our density profiles in Fig. 2 for the trapped case and the spatial dependence of the gap $\Delta_0(\mathbf{r})$ as defined in section 4 we can show that the main source of disagreement between our theoretical model and the experimental data is due to an incorrect modeling of the normal state cloud. The boundary of the superfluid region is determined by the condition $\Delta_0(\mathbf{r}) = \Delta_{0,c}$; as already explained in section 4, $\Delta_{0,c}$ is determined by comparing the free energies of the normal state and of the superfluid state. This quantity alone determines the key features of a trapped configuration of a Fermi gas and is frequently used in comparing different theories and experimental data[42]; our model predicts $\Delta_{0,c} \approx 1.4\zeta$ across the whole crossover, while experimental observations and Montecarlo simulations[23, 42] at unitarity suggest that the superfluid phase breaks down approximately for $\Delta_{0,c} \approx \zeta$.

The origin of this disagreement is better understood by thinking in terms of free energy: we model the normal state as a non-interacting Fermi gas; by modeling the fermion-fermion interaction in the normal part we would lower the energy there, making the normal state energetically favorable in a bigger spatial range. By noting that $\Delta_0(\mathbf{r})$ is monotonically decreasing as the distance from the center of the trap increases, it follows that pushing the phase boundary closer to the trap center effectively increases $\Delta_{0,c}$. Therefore we conclude that the mean-field approximation of the normal state directly affects the polarization values we report. On the other hand, the mean-field approximation of the SF core has been shown to be much safer: other artifacts could arise as we neglect the fluctuation contribution to the condensate fraction in the SF core, but this term has been shown to give much smaller contributions to our final result[53], well below experimental sensitivities.

At last we take into account the possibility of an Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state, i.e. a grand potential minimum competing with the BCS and normal phases, which has been shown to be a global minimum for an adequate choice of parameters [45, 46, 31]. The existence of such a state in a three dimensional system is still subject to intense research: the stability of the FFLO state in a three dimensional system has been questioned (see for instance [60] and citations therein), and such a state has not yet been observed in 3D systems[57]. Eventually, the FFLO state occupies a very small region in the parameters space, as reported in [58, 60], and would not affect our theoretical estimates significantly larger than the other sources of error we analyzed. Means of enhancing the FFLO state in 3D which could allow an experimental observation have been proposed[57, 59] on theoretical grounds, but up to date they have not been implemented. As the main aim of the present paper is to compare our theoretical model with the experimental data in [7], as the authors in [7] reported

that they did not observe a modulation in the condensate density as predicted by the theory of the FFLO state, we decided not to include the FFLO state in our theoretical description of the unbalanced Fermi gas.

6. Conclusions

We have considered a two-component polarized Fermi gas in three dimensions across the BCS-BEC crossover. Starting from a path integral formulation, we have derived a formula for the condensate fraction at finite temperature. In the limit of zero temperature, this system has been analyzed at mean-field level (MF), both in the absence and in the presence of an external trapping which has been dealt with in the local density approximation (LDA). The condensate fraction has been studied as a function of the interatomic strength and polarization. We have discussed how an enhancement of the population imbalance produces a depletion of the condensate fraction which is shown to decrease linearly in the deep BEC regime. We have, finally, analyzed the harmonically-trapped condensate and the total density profiles in correspondence to various interaction parameters by commenting that the finite discontinuity between the two densities at the phase boundary is a consequence of the phase separation. The comparison of our results with the experimental data of Zwierlein *et al.* [42] has shown that our predictions are in reasonable agreement with the experimental data only in the BCS side of the crossover up to unitarity and for moderate polarization values.

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